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SERVICE LIFE OF PRESSURIZED VESSELS

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A study is made of the dependence of the time of propagation of a fracture front in thick-walled vessels subjected to internal pressure on the wall thickness and the employed version of kinetic theory of creep. The stress intensity is taken to be the equivalent stress. Recommendations on using the results in computational applications are formulated.

Key words: *creep, damage parameter, fracture onset time, pressurized vessels, fracture front propagation.*

In computation practice, the time of deformation to fracture for a body of an arbitrary configuration (a structural member) is usually divided into two intervals [1] which correspond to different stages of material deformation under external thermal-force loading. The first stage, characterized by damage accumulation, ends when the accumulated damage reaches a critical value at a certain point or region of the body, which corresponds to the onset of fracture. The second stage, characterized by the propagation of the fracture front, is finished by complete fracture of the body (for example, by its fragmentation).

The estimation of the time of propagation of the fracture front is a labor-consuming problem because neither the front shape nor the direction of its propagation are known in advance. Apparently, for this reason, the life of a body is sometimes identified with the time of onset of its fracture [1], i.e., with the duration of the first stage of deformation (the latent fracture stage).

Estimation of the time of propagation of the fracture front has been the subject of a small number of studies [2–10]. The results of these studies are sometimes contradictory. For example, in studies of the propagation of the fracture front in a thick-walled tube under internal pressure, it has been established that its duration can be about 50% of the duration of the stage of latent fracture, i.e., the tube with partially fractured cross section can resist external loading for a long time [3]. In a similar problem for a rotating disk, it is noted that fracture occurs catastrophically rapidly [5] (the duration of this stage is about 6–7% of the duration of the stage of latent fracture).

It is obvious that such contradictory estimates of the time of propagation of the fracture front are due to several reasons, including the use of various versions of creep theory, which are not always equivalent, and different criteria of creep rupture strength and objects of research (thick-walled tubes [3, 4, 8–10], rotating disks [5, 6], extended plates of constant thickness [3, 7], beams in straight transverse bending [3], and torque shafts [2]).

In the present paper, we study the dependence of the duration of fracture front propagation in thick-walled vessels under internal pressure on the wall thickness and the version of kinetic creep theory used. The stress intensity that coincides with the maximum tangential stress to within a factor is taken to be the equivalent stress.

1. Stage of Latent Fracture. Let a uniformly heated thick-walled vessel having the shape of a tube in rectangular cylindrical coordinates (r, φ, z) or the shape of a sphere in spherical coordinates (r, φ, ψ) be subjected to a stationary internal pressure q. Any point of the tube, the stresses σ_φ, σ_z, and σ_r arise, and at any point of the sphere, the stresses σ_φ, σ_ψ, and σ_r arise, all of them being principal. At any time t, the stresses σ_φ and σ_r satisfy the equilibrium equation

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$$\frac{d\sigma_r}{dr} + \frac{s(\sigma_r - \sigma_\varphi)}{r} = 0 \tag{1}$$

and the boundary conditions

$$\sigma_r(a_0, t) = -q, \qquad \sigma_r(b_0, t) = 0, \tag{2}$$

where a_0, b_0, and r are the inner, outer, and current radii of the vessel; $s = 1$ for the tube and $s = 2$ for the sphere.

The principal stress vector σ_z is counterbalanced by the internal pressure force on the tube bottom:

$$2\int_{a_0}^{b_0} \sigma_z r\, dr = a_0^2 q.$$

For the sphere, by virtue of symmetry, $\sigma_\varphi = \sigma_\psi$.

Let the tube be in a plane strain state ($\dot{p}_z = 0$), and in the sphere, by virtue of symmetry, $\dot{p}_\varphi = \dot{p}_\psi$. From the aforesaid, the incompressibility condition for the material is written as $s\dot{p}_\varphi + \dot{p}_r = 0$. Here p_φ, p_ψ, p_r, and p_z are the circumferential, meridional, radial, and axial strain components, respectively; the dot denotes the time derivative. Because $\dot{p}_\varphi = \dot{u}/r$ and $\dot{p}_r = d\dot{u}/dr$, it follows that $\dot{u}(r,t) = c_1(t)/r^s$ [$u(r,t)$ is the radial displacement of an arbitrary point of the Vessel and $c_1(t)$ is a function of time]. In view of the aforesaid, we obtain

$$\dot{p}_\varphi = c_1(t)/r^{1+s}, \qquad \dot{p}_r = -sc_1(t)/r^{1+s}. \tag{3}$$

As physical relations we use the Rabotnov kinetic theory of creep. The relations of this theory are written as [5, 11]

$$\dot{p}_{ij} = \frac{W}{\sigma_{\rm e}}\frac{\partial\sigma_{\rm e}}{\partial\sigma_{ij}}, \qquad W = \frac{B_1\sigma_{\rm e}^{n+1}}{\varphi_1(\omega)}, \qquad i,j = 1,2,3; \tag{4}$$

$$\dot{\omega} = \frac{B_2\sigma_{*\rm e}^{g+1}}{\varphi(\omega)}, \qquad \omega(x_k, 0) = 0, \qquad \omega(x_k^*, t_*) = 1, \tag{5}$$

where p_{ij} and σ_{ij} are the creep strain and stress tensor components; $\sigma_{\rm e}$ and $\sigma_{*\rm e}$ are the equivalent stresses, which are homogeneous first-order functions of stress, $W = \sigma_{ij}\dot{p}_{ij}$ is the specific energy dispersion power for creep of the material, B_1, n, B_2, and g are the creep and creep rupture strength characteristics of the material, ω is a parameter that describes the creep damage accumulation from a phenomenological standpoint, and t_* is the time of fracture onset [the moment at which the damage parameter first reaches a critical value equal to unity at a certain point with the coordinates x_k^* ($k = 1,2,3$)].

It is obvious that system (4), (5) is a fairly general version of the kinetic theory. If $\varphi_1(\omega) = \varphi(\omega)$, one obtains the most widespread version of this theory, whose particular cases are short-term creep theory [5, 12], the energy version of creep and creep rupture strength theory [11, 13, 14], and various modifications of hardening theory [5, 14]. If $\varphi_1(\omega) = 1$, one obtains a simplified version of the Kachanov kinetic theory of creep [2, 3]. We will assume that

$$\varphi_1(\omega) = (1-\omega)^{m_1}, \qquad \varphi(\omega) = (1-\omega)^m, \qquad 0 \leqslant m_1 \leqslant m. \tag{6}$$

To simplify the results, as the equivalent stresses in (4) and (5) we use the stress intensities $\sigma_i = \lambda_1(\sigma_\varphi - \sigma_r)$ ($\lambda_1 = \sqrt{3}/2$ for the tube and $\lambda_1 = 1$ for the sphere). In this case, under the assumptions made above, system (4) becomes

$$\dot{p}_\varphi = \frac{\lambda_1^{n+1}B_1}{s}\frac{(\sigma_\varphi - \sigma_r)^n}{\mu^{m_1}}, \qquad \dot{p}_r = -\lambda_1^{n+1}B_1\frac{(\sigma_\varphi - \sigma_r)^n}{\mu^{m_1}}, \tag{7}$$

where μ is an analog of the Kachanov continuity parameter:

$$\mu(r,t) = [1 - \omega(r,t)], \tag{8}$$

and, according to (6) and (8), the functions φ_1 and φ are defined as $\varphi_1(\mu) = \mu^{m_1}$ and $\varphi(\mu) = \mu^m$. The substitution (8) transforms Eq. (5) to the equation

$$\mu^m\dot{\mu} = -B_2\sigma_i^{g+1}, \qquad \mu(r,0) = 1, \qquad \mu(r^*, t_*) = 0, \tag{9}$$

which implies that

$$\mu(r,t) = \Big(1 - (m+1)\lambda_1^{g+1} B_2 \int_0^t (\sigma_\varphi - \sigma_r)^{g+1}\, d\tau\Big)^{1/(m+1)}. \tag{10}$$

System (1)–(3), (7), (9), with the use (6) and (8), allows one to calculate the stress–strain state at any time with allowance for damage accumulation in the material of uniformly heated thick-walled vessels loaded by internal pressure.

The procedure of this calculation and its results are given below. Comparing (3) and (7), we obtain $\sigma_\varphi - \sigma_r$ and $\sigma_i = \lambda_1(\sigma_\varphi - \sigma_r)$:

$$\sigma_\varphi - \sigma_r = \frac{q}{sJ_1}\, \frac{[\mu(r,t)]^{m_1/n}}{X(t)}\, r^{-(1+s)/n}, \qquad \sigma_i = \frac{\lambda_1 q}{sJ_1}\, \frac{[\mu(r,t)]^{m_1/n}}{X(t)}\, r^{-(1+s)/n}. \tag{11}$$

Substituting (11) into (9) and (1), from (9) we have

$$\int_1^\mu z^k\, dz = -[(m+1)t^0]^{-1} \int_0^t [X(\tau)]^{-(g+1)}\, d\tau, \qquad k = [mn - m_1(g+1)]/n. \tag{12}$$

In view of the first boundary condition in (2), relation (1) implies that

$$\sigma_r(r,t) = -q + \frac{q}{J_1} \int_{a_0}^r \frac{\mu^{m_1/n}}{X(t)}\, r^{-1-(1+s)/n}\, dr, \tag{13}$$

and the second boundary condition in (2) implies that

$$\int_{a_0}^{b_0} [\mu(r,t)]^{m_1/n} r^{-1-(1+s)/n}\, dr = J_1 X(t). \tag{14}$$

In (11)–(14),

$$J_1 = \int_{a_0}^{b_0} r^{-1-(1+s)/n}\, dr, \qquad X(t) = \frac{q}{sJ_1}\Big(\frac{sc_1(t)}{\lambda_1^{n+1} B_1}\Big)^{-1/n}; \tag{15}$$

$$t^0(r) = \Big[(m+1) B_2 \lambda_1^{g+1} \Big(\frac{q}{sJ_1}\Big)^{g+1} r^{-(1+s)(g+1)/n}\Big]^{-1}. \tag{16}$$

Obviously, solution of the problem formulated above reduces to solution of system (12), (14) for $\mu(r,t)$ and $X(t)$. Integration of (12) yields

$$[\mu(r,t)]^{m_1/n} = \Big(1 - \frac{\bar\nu}{t^0} \int_0^t [X(\tau)]^{-(g+1)}\, d\tau\Big)^{\bar\beta}, \tag{17}$$

where

$$\bar\beta = \frac{m_1}{n + nm - m_1(g+1)}, \qquad \bar\nu = \frac{n + nm - m_1(g+1)}{n(m+1)}. \tag{18}$$

Substitution of (17) into (14) yields the following equation for $X(t)$:

$$\int_{a_0}^{b_0} \Big(1 - \frac{\bar\nu}{t^0} \int_0^t [X(\tau)]^{-(g+1)}\, d\tau\Big)^{\bar\beta} r^{-1-(1+s)/n}\, dr = J_1 X(t). \tag{19}$$

Knowing $\mu(r,t)$ and $X(t)$, it is possible to determine the stress–strain state of the pressurized vessels. Indeed, from (13) we determine $\sigma_r(r,t)$, and from (11), we determine $\sigma_\varphi(r,t)$ and $\sigma_i(r,t)$. The fracture onset time is calculated from the condition $\mu(r^*, t_*) = 0$. It is easy to show that fracture begins on the inner surface of the vessel ($r^* = a_0$). For the integral in (12) to converge in the case $\mu(r^*, t_*) = 0$, the material characteristics need to be subjected to

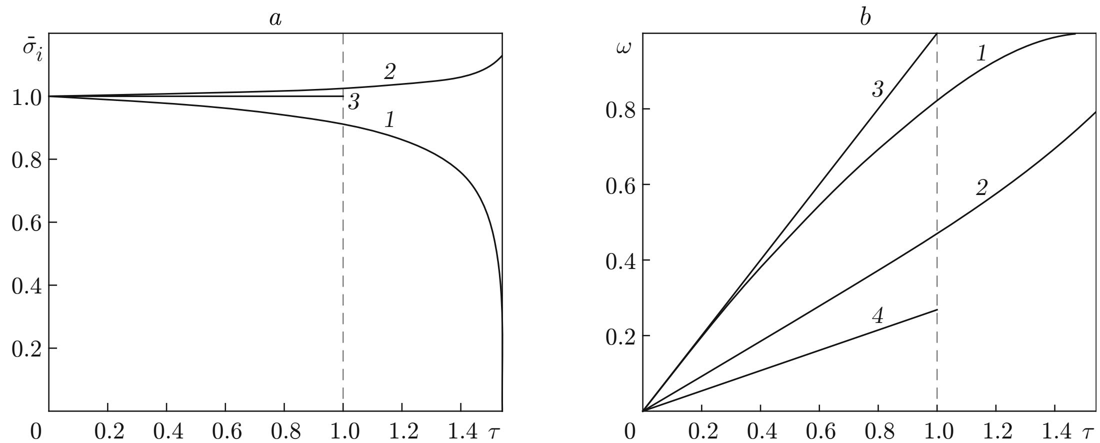

Fig. 1. Normalized stress $\bar{\sigma}_i$ (a) and damage parameter ω (b) versus normalized time τ for the inner (1 and 3) and outer (2 and 4) radii of the tube: curves 1 and 2 refer to the calculation results obtained in the present work and curves 3 and 4 refer to the calculation results obtained using the Kachanov theory; the vertical dashed curves are the fracture onset time according to the Kachanov theory.

constraints. Without writing these constraints, we note that they agree with experimental data [11, 14]. The strain state is found from (3) through the use of (15).

The case $m_1 = 0$ corresponds to calculations using the Kachahov kinetic theory. In this case, the stress state, which we denote by $\sigma_i^0(\rho)$, is stationary. The fracture onset time will be denoted by t_*^0. This time is calculated from (16) for $r = a_0$, i.e., $t_*^0 = t^0(a_0)$.

Figure 1 gives curves of the normalized stress $\bar{\sigma}_i = \sigma_i(\rho, \tau)/\sigma_i^0(\rho)$ and the damage parameter ω versus the normalized time $\tau = t/t_*^0$ in tube sections for various values of the current radius $\rho = r/a_0$ ($1 \leqslant \rho \leqslant \beta_1$ and $\beta_1 = b_0/a_0$). Curves 1 correspond to the redistribution of the chosen quantities on the inner radius ($\rho = 1$), and curves 2 refers to the outer radius ($\rho = \beta_1$ and $\beta_1 = 2$).

In the calculations, we used the following constants in the constitutive equations (4)–(6): $B_1 = 3.5 \times 10^{-15}$ MPa$^{-n} \cdot$ h^{-1}, $B_2 = 2.8 \cdot 10^{-15}$ MPa$^{-(g+1)} \cdot$ h^{-1}, $n = 6$, $g = 4.75$, $m = 14$, $m_1 = 10$, and $q = 40$ MPa. The values of the indicated characteristics correspond to D16T alloy at a temperature 250°C [13, 14].

From the data presented in Fig. 1 it follows that fracture begins on the inner surface. Here the damage parameter reaches the critical value equal to unity and the normalized stress decreases to zero. At all points of the volume of the thick-walled vessels there is a continuous redistribution of the stress field, which, as noted above, does not follow from the Kachanov theory solution (curve 3 in Fig. 1a), which exists in the interval $0 \leqslant \tau \leqslant 1$ (vertical dashed line). The variations in the damage parameter of this solution on the inner and outer radii are shown in Fig. 1b (curves 3 and 4). From Fig. 1 it also follows that the fracture onset time exceeds (by more than 1.5 times in the given example) the time obtained from the Kachanov theory solution, and depends greatly on the wall thickness of the vessel β_1 and the material characteristic m_1.

Figure 2a gives a family of curves whose parameter is the tube wall thickness ($\tau_* = t_*/t_*^0$ is the normalized fracture onset time and m_1 is the material characteristic). Figure 2b shows the distribution of the parameter μ in a section of the tube at the time of fracture onset versus the material characteristic m_1. At the initial time, the material is not damaged (curve 1). Curves 2–4 are the distribution of the parameter μ over the section ($1 \leqslant \rho \leqslant \beta_1$ and $\beta_1 = 2$) at the time of fracture onset ($t = t_*$) for various values of m_1. In practice, life is sometimes estimated by the fracture onset time t_* [1]. We assume that the expended life is determined by the damage accumulated in the material. From Fig. 2b it follows that the residual life of the vessel increases with decreasing material characteristic m_1. The results show that the Kachanov kinetic theory of creep predicts unreasonably underestimated life of the vessel compared to the Rabotnov kinetic theory of creep. Thus, the life of the vessel is the longer the larger the value of m_1, which can be used as the basis for an optimum choice of the vessel material for operation in a specified temperature loading range. Therefore, it is reasonable to estimate the time of propagation of the fracture front.

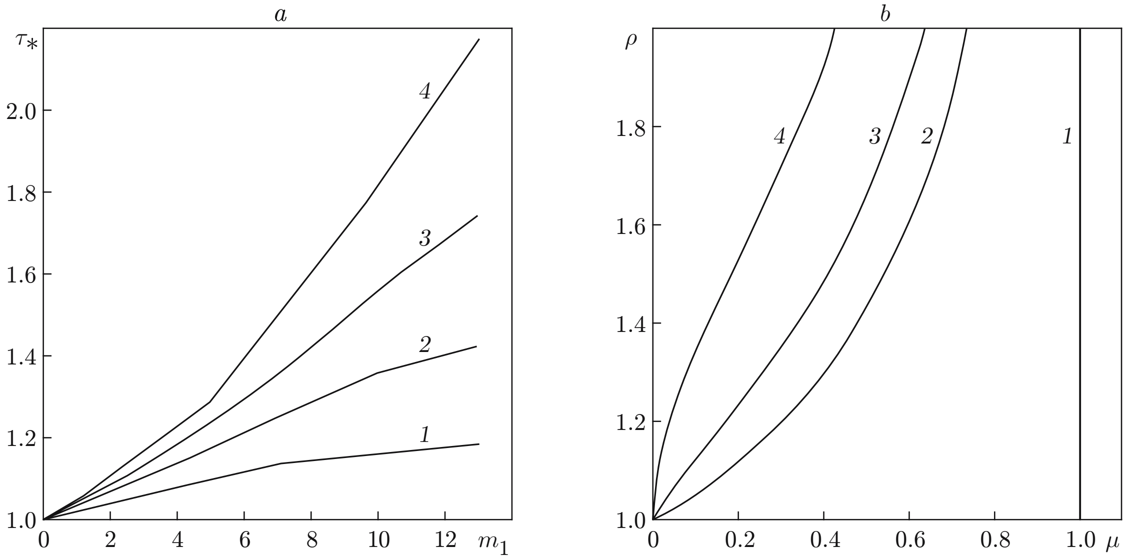

Fig. 2. Time of fracture onset in the tube (a) and residual life (b) versus the material characteristic m_1: (a) $\beta_1 = 1.2$ (1), 1.5 (2), 2 (3), and 3 (4); (b) $t = 0$ (1) and $m_1 = 0$ (2), 5 (3), and 10 (4).

2. Stage of Propagation of Fracture Front. It should be noted that the solution of the problem of fracture front propagation in a body of arbitrary configuration is considerably complicated since the site of origin of the front, its shape, and propagation direction are not known in advance.

At $t > t_*$, two zones are formed in the body: a zone where $1 > \mu(t) > 0$ and a zone where fracture has already occurred and the material has lost the ability to resist [5]. The boundary between the fractured and intact zones is the fracture front, which moves in such a manner that, at the fracture front, $\mu = 0$ and no forces from the fractured zone are transferred to the intact zone [5].

It should be noted that the fracture onset site at the time $t = t_*$ depends greatly on the choice of the equivalent stress. In the case of thick-walled tubes, fracture can begin on the outer surface $r = b_0$ if the largest normal stress is used as the equivalent stress [3], it can begin on the inner surface $r = a_0$ when using the stress intensity adopted in [10] or on the surface $r = c$, $a_0 < c < b_0$ if the equivalent stress is the product of power functions of the stress intensity and the largest normal stress [8]. In the problem considered, fracture arises on the inner surface $r = a_0$ of the pressurized vessels, the fracture front is symmetric and moves toward the outer surface $r = b_0$, and the radius of the fracture front $r = a(t)$ at any time $t > t_*$. It is obvious that $a(t_*) = a_0$ and $a(t_{*f}) = b_0$ (t_{*f} is the time corresponding to the complete fracture of the vessel). The time of propagation of the fracture front is $t_{*f} - t_*$.

In the intact zone, where $1 > \mu(t) > 0$, Eqs. (3)–(10) and the equilibrium equation (1) are valid at any time $t > t_*$ and, in view of the above remarks, boundary conditions (2) are written as

$$\sigma_r(a(t), t) = -q, \qquad \sigma_r(b_0, t) = 0.$$

By analogy with (11) and (15), we obtain

$$\sigma_\varphi - \sigma_r = \frac{q}{sJ_{1f}} \frac{[\mu(r,\tau)]^{m_1/n}}{X(\tau)} r^{-(1+s)/n}, \qquad \sigma_i = \lambda_1(\sigma_\varphi - \sigma_r); \tag{20}$$

$$J_{1f} = \int\limits_{a(t)}^{b_0} r^{-1-(1+s)/n}\, dr. \tag{21}$$

In (20) and (21) and in the following, $a(t) \leqslant r \leqslant b_0$. The replacement of J_1 by J_{1f} does not disturb the structure of relations (15) and (16).

TABLE 1

Relative Time of Propagation of the Fracture Front in the Tube $\bar{\tau}_*$

β_1	m_1	C/A	$\bar{\tau}_*$	C/B
1.2	0	0.302	0.053	0.053
	3	0.176	0.036	0.036
	5	0.105	0.025	0.024
	7	0.050	0.014	0.014
	10	0.0067	0.0028	0.0028
	13	0.000017	0.000015	0.000015
2	0	0.824	0.214	0.214
	3	0.625	0.178	0.177
	5	0.480	0.149	0.147
	7	0.331	0.115	0.113
	10	0.123	0.057	0.056
	13	0.008	0.007	0.007
4	0	1.254	0.467	0.467
	3	1.051	0.415	0.412
	5	0.890	0.372	0.367
	7	0.704	0.318	0.312
	10	0.385	0.212	0.206
	13	0.079	0.071	0.070

TABLE 2

Relative Time of Propagation of the Fracture Front in the Sphere $\bar{\tau}_*$

β_1	m_1	C/A	$\bar{\tau}_*$	C/B
1.5	0	0.758	0.186	0.186
	3	0.562	0.152	0.151
	5	0.423	0.125	0.124
	7	0.282	0.095	0.093
	10	0.097	0.044	0.043
	13	0.005	0.004	0.004
2	0	1.052	0.336	0.336
	3	0.849	0.291	0.289
	5	0.693	0.254	0.251
	7	0.521	0.209	0.206
	10	0.247	0.126	0.122
	13	0.033	0.030	0.029
2.5	0	1.247	0.462	0.462
	3	1.044	0.411	0.408
	5	0.883	0.367	0.363
	7	0.698	0.314	0.308
	10	0.380	0.208	0.203
	13	0.077	0.069	0.068

Using (20), (21), (16) and (18), instead of (17) we obtain the expression

$$[\mu(r,t)]^{m_1/n} = \Big(1 - \bar{\nu} c r^{-(1+s)(g+1)/n} \int\limits_0^t [J_{1f}(\tau)X(\tau)]^{-(g+1)}\, d\tau\Big)^{\bar{\beta}}, \tag{22}$$

where $c = (m+1)B_2(\lambda_1 q/s)^{g+1}$.

At the time t, let the radius of the fracture front be a. On the line of the fracture front, $\mu(a(t),t) = 0$. In view of this, from (22) we have

$$a^{(1+s)(g+1)/n} = \bar{\nu} c \int\limits_0^t [J_{1f}(\tau)X(\tau)]^{-(g+1)}\, d\tau. \tag{23}$$

Differentiation of (23) with respect to t yields

$$a^{-1+(1+s)(g+1)/n} \frac{da}{dt} = \frac{n\bar{\nu}c}{(1+s)(g+1)} \Big[J_{1f}(t)X(t)\Big]^{-(g+1)}. \tag{24}$$

Using (23), from (22) we obtain

$$[\mu(r,t)]^{m_1/n} = \Big[1 - (r/a(t))^{-(1+s)(g+1)/n}\Big]^{\bar{\beta}}. \tag{25}$$

Replacing a_0 by $a(t)$ and J_1 by J_{1f} in (14) and substituting relation (25) into the expression obtained, instead of (19) we have

$$\int\limits_{a(t)}^{b_0} \Big[1 - \Big(\frac{r}{a}\Big)^{-(1+s)(g+1)/n}\Big]^{\bar{\beta}} r^{-1-(1+s)/n}\, dr = J_{1f}(t)X(t). \tag{26}$$

Integration of (24) with the use of (26) yields

$$I(a) = \frac{\bar{\nu}cn}{(1+s)(g+1)} \int\limits_{t_*}^t d\tau,$$

$$I(a) = \int\limits_{a_0}^{a} a^{-1+(1+s)(g+1)/n} \Big\{\int\limits_a^{b_0} \Big[1 - \Big(\frac{r}{a}\Big)^{-(1+s)(g+1)/n}\Big]^{\bar{\beta}} r^{-1-(1+s)/n}\, dr\Big\}^{g+1} da. \tag{27}$$

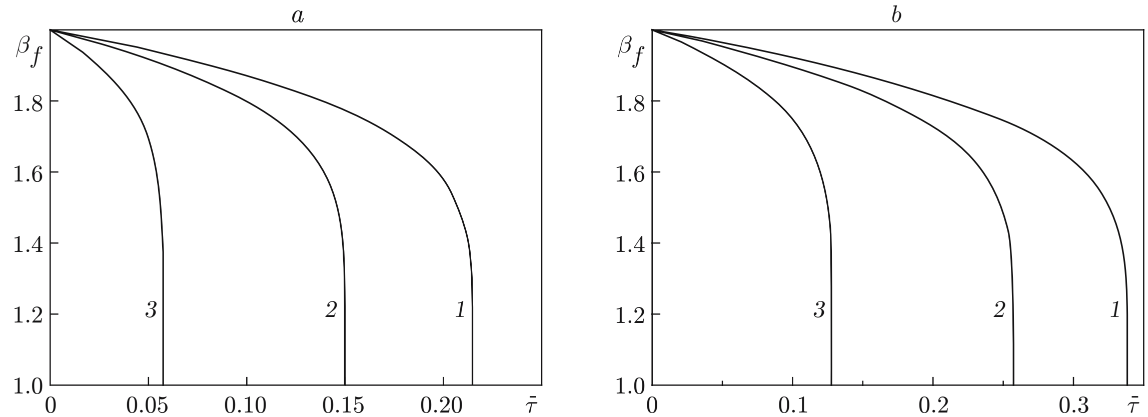

Fig. 3. Variation in the wall thickness of the tube (a) and sphere (b) during propagation of the fracture front: $m_1 = 0$ (1), 5 (2), and 10 (3).

In the integration in (27), it is reasonable to convert to the new variables $v(t) = a(t)/a_0$ and $u(t) = r/a(t)$. Obviously, $v(t_*) = 1$ and $v(t_{*f}) = \beta_1$; $u(t) = 1$ for $r = a(t)$ and $u(t) = \beta_1/v(t)$ for $r = b_0$.

Tables 1 and 2 give values of the time $\bar{\tau}_* = (t_{*f} - t_*)/t_*$ and its upper and lower estimates for various values of β_1 and m_1 for the tube and sphere. These results show that the values of $\bar{\tau}_*$ increase with increasing wall thickness of the vessel for any fixed value of m_1 and decrease with increasing m_1 for any wall thickness of the vessel.

As an example, Fig. 3 gives results of solution of Eq. (27) for the tube and sphere [$\bar{\tau} = (t - t_*)/t_*$ and β_f is the thickness of the intact part of the vessel during fracture front propagation]. At the moment the front begins to move, $\beta_f = \beta_1$ ($\beta_1 = b_0/a_0$), and with time it decreases to unity, which corresponds to the complete fracture of the vessel material. At the initial time $t > t_*$, the front moves rather slowly, and then it moves catastrophically rapidly irrespective of the material characteristic m_1 and the vessel wall thickness.

An analysis of the results leads to the conclusion that the time of propagation of the fracture front can be a few hundredth to tens of percent of the latent fracture time t_*, depending on the vessel wall thickness and the material characteristic m_1. This should be taken into account in computation practice. It is obvious that the identification of the life of a structural member with the fracture onset time, as is done in practice [1], is not always correct. Therefore, it seems reasonable to obtain upper and lower estimates of the time of fracture front propagation in pressurized thick-walled vessels.

We note that the result of integration of (27) can be represented in compact form if the time of fracture front propagation $t_{*f} - t_*$ is normalized not to t_* but to t_*^0. Using this expression and the corresponding expressions for J_1, t_*^0, and c from (27), we finally obtain

$$\frac{t_{*f} - t_*}{t_*^0} = \frac{(1+s)(g+1)}{\bar{\nu} n} f(\beta_1) I(\beta_1), \tag{28}$$

where

$$I(\beta_1) = \int_1^{\beta_1} v^{-1} \Big[\int_1^{\beta_1/v} u^{-1-(1+s)/n} \Big(1 - u^{-(1+s)(g+1)/n}\Big)^{\bar{\beta}} du \Big]^{g+1} dv,$$

$$f(\beta_1) = \Big(J_1 a_0^{(1+s)/n}\Big)^{-(g+1)} = \Big(\frac{1+s}{n}\Big)^{g+1} \Big(\frac{\beta_1^{(1+s)/n}}{\beta_1^{(1+s)/n} - 1}\Big)^{g+1}.$$

The upper and lower estimates of the fracture onset time of structural members were obtained in [15]:

$$A \le \frac{t_*}{t_*^0} \le B, \qquad A = \frac{1 - (1-\lambda)^{g+2}}{\lambda \bar{\nu} (g+2)}, \qquad B = \frac{1 - (1-\lambda)^{1/\bar{\nu}}}{\lambda}, \qquad \bar{\beta} < 1,$$
$$\bar{\beta} = \frac{m_1}{n + mn - m_1(g+1)}, \qquad \bar{\nu} = \frac{n + mn - m_1(g+1)}{n(m+1)}, \qquad \lambda = \frac{t_*^0}{\bar{t}^0}. \tag{29}$$

Here $\bar{t}^0$ is given by the relation

$$\int\limits_V \frac{W^0}{t^0}\,dV = \frac{1}{\bar{t}^0}\int\limits_V W^0\,dV,$$

where W^0 is the energy dissipation power under the assumption of steady-state creep. In the case $\bar{\beta} > 1$, the boundaries of the variation of the quantity t_*/t_*^0 in (29) are interplaced.

Using inequalities (29) together with (28), we finally obtain

$$\frac{C}{B} \le \frac{t_{*f} - t_*}{t_*} \le \frac{C}{A}, \tag{30}$$

where C is the right side in (28).

In Tables 1 and 2, numerical results of the relative time of fracture front propagation in pressurized vessels are compared with its lower and upper estimates according to inequalities (30). (The data in Tables 1 and 2 are given for $\bar{\beta} < 1$.)

From the analysis of the results it follows that the fracture onset time and the time of propagation of the fracture front depend greatly on the vessel wall thickness and the material characteristic m_1. The estimates (30) are recommended for use in computation practice.

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